Introduction to Economic Scenario Generators

Jon Mossman FCIA, CFA
Agenda

1. Introduction to ESGs
2. Real World vs. Market Consistent ESG
3. Basics
4. Market Consistent ESG
5. Real World ESG
6. Practical Modelling Issues
7. Q & A
What is modelling?

That takes known data “Inputs”

And constructs a process that maps inputs to answers “modelling”

Questions we want answered “Outputs”

If the inputs are uncertain…

A range of outputs is possible

And the model will be referred to as stochastic
What is modelling?

1. What is an ESG
1. What is an ESG

**What is an Economic Scenario Generator (ESG)?**

- Model that projects the value of economic values (e.g., stock returns, interest rates, corporate bond spreads, property values) into the future
- Models can be deterministic or stochastic
- A stochastic model will project a distribution of possible future values

![Distribution of stock values](chart.png)
1. What is an ESG

What is an Economic Scenario Generator (ESG)?

- Output is a time-series of variables for each scenario (trial)
- Monthly, quarterly, semi-annual, annual
Why Do We Need ESG?

- Real World:
  - Decision making under uncertainty
  - Financial markets are unpredictable
  - Variability exhibits structure and shape

- Risk Neutral
  - Correctly pricing/valuing contingent cash flows
What is the ESG doing?

1. What is an ESG

Assets required

Market / target

Calibration

Model of the Market / target

Scenario generation

Scenarios output

Simulation of the Model of the Market / target
Different ESGs answer different questions

Question:

- How extreme can my losses get?
- Projects looking at the “range of possible answers”
- Risk based capital projects

A good model is one that produces a “good distribution of future outcomes”

- Mean outcome, volatility, skewness, correlations

Referred to as “real world scenarios”
Question: What is the value of the cashflows at a set date?
Projects looking at “value”
EEV / pricing / fair value projects

A good model is one that captures value consistently.

Models that do not permit arbitrage and are calibrated to fit market prices - “market-consistent” scenarios
Recap: Specify the problem you are solving

- What do you need from the scenarios?
- What is the *purpose* of the modelling exercise?
- Do you want to:
  - Place a *value* on a set of cashflows?
    - An arbitrage free valuation that reflects *current market* prices?
  - Reflect management views as to how a set of cashflows will *evolve*?

1. What is an ESG
1. What is an ESG

How ESGs work

PAST


FUTURE
1. What is an ESG

Market-consistent scenarios

Past Unimportant

Markets do not permit arbitrage

Projection fitting asset prices

Asset Prices

© 2006 Towers Perrin
1. What is an ESG

**Real-world scenarios**

- Research / historic data
- Management views on the future

PAST

- 1986
- 1996

FUTURE

- 2007
- 2016

Projection

Analysis of historic data
Agenda

1. Introduction to ESGs
2. Real World vs. Market Consistent ESG
3. Basics
4. Market Consistent ESG
5. Real World ESG
6. Practical Modelling Issues
7. Q & A
Both market-consistent and real-world scenarios have value for ALM

- Real-world scenarios tell you the risk/reward tradeoffs of bearing risk
  - These scenarios are used to get distributions of expected results
  - Real-world scenarios are used to determine risk measures such as VaR and CTE
Both market-consistent and real-world scenarios have value for ALM

- Market-consistent scenarios tell you the market-price of a liability
  - This is the cost for getting rid of (hedging) the liability in the capital markets
  - It can serve as a check on whether a liability is being mis-priced using real-world scenarios
- Since assets are valued on a market-consistent basis (by definition!), calculating risk measures (e.g., duration, convexity) using these scenarios provide a consistent benchmark to use for asset/liability management

Continued…
Some ESG terminology

- **Arbitrage-free**
  - Efficient markets do not permit risk-free profits
  - Replicating portfolios have the same price
  - Applies to both real-world and market-consistent scenarios

- **Risk-neutral**
  - A state in which investors do not require a premium for assuming risk
  - Assets are priced by calculating expected values over each scenario path and weighting each path by its risk-neutral probability

Continued…
Some ESG terminology

- Market-consistent
  - Scenarios that replicate current asset prices
  - Scenarios are both arbitrage-free and risk-neutral, plus have market-based expected returns and volatility

Frequently, “risk-neutral” and “arbitrage-free” terms are used when, in fact, “market-consistent” is intended.
The average return follows the rates embedded in the initial risk-free yield curve.

All assets (even equities) have an “expected” return equal to the risk-free rate.

It may be helpful to view this as a risk-averse investor giving greater weight to the adverse scenario.

Projected interest rate yield curves flatten on average and have many inversions.

Some interest scenario generators produce negative interest rates, although this is a consequence of the model type and not necessary for risk neutrality.
Volatilities are calibrated from asset prices — these “implied volatilities” may appear unrealistic compared with long term historical experience.

- Will fluctuate over time (as can actual volatility)
- Will vary by type of asset, e.g., out-of-money derivatives may have higher implied volatility than in-the-money derivatives
- Customized option from bank may have distorted volatility based on bank’s loading

On a positive note, once a model structure is set, the expected return and volatilities are determined from actual asset prices. No subjective input is needed.
Real-world scenarios place a burden on the user to make subjective decisions about the future

- Some of these decisions may be related to historical data
  - Short-term volatility, by nature, has a large number of historical data points
  - Some characteristics, like correlation between long and short-term rates, may be non-controversial

- Longer-term assumptions are highly subjective
  - E.g., how many 10-year periods have there been in the past 20 years? Data from long-ago is from an economic/political environment different than today
  - Key long-term assumptions are whether and how quickly rates revert to a long-term mean. What is the mean? Is the mean constant?
Recap: Definitions

- **RISK NEUTRAL SCENARIOS**
  - Aim: valuation that does not allow arbitrage \textit{but is not} necessarily consistent with market prices

- **MARKET CONSISTENT SCENARIOS**
  - Aim: valuation that does not allow arbitrage \textit{and} is calibrated to produce values consistent with market prices

- **REAL WORLD SCENARIOS**
Agenda

1. Introduction to ESGs
2. Real World vs. Market Consistent ESG
3. Basics
4. Market Consistent ESG
5. Real World ESG
6. Practical Modelling Issues
7. Q & A
3. Basics

**Brownian Motion**

- $\Delta B_t = Z\sqrt{\Delta t}$
- $Z \sim N(0, 1)$
- $B_t \sim N(0, \sqrt{t})$
- $\Delta B_t \sim N(0, \sqrt{\Delta t})$
3. Basics

Generalized Weiner Process

- $\Delta X_t = a \Delta t + b \Delta B_t$
- Drift = $a$ per unit of time
- Variance = $b$ per unit of time
Stochastic Differential Equations

- $dR_t = \mu(R_t, t) \, dt + \sigma(R_t, t) \, dB_t$
- If $\mu, \sigma$ constant then Generalized Weiner Process
- If a function of $R$ and $t$ then Ito process
3. Basics

Geometric Brownian Motion

- $\Delta S_t = \mu S_t \Delta t + \sigma S_t \Delta B_t$
- $\Delta \ln(S_t) = (\mu - \sigma^2/2) \Delta t + \sigma \Delta B_t$
- $S_t = S_0 \exp[(\mu - \sigma^2/2) t + \sigma B_t]$
3. Basics

Key questions

- Choice of model
- Calibration of the model
- How good are the scenarios?
- How many scenarios should we use?
- Better scenarios
3. Basics

**Does the choice of model matter?**

- There are many different mathematical formulations. For interest rate projections ...
  - Vasicek
  - Hull White
  - CIR

- In theory
  - the choice of model *does not matter as long as the model is calibrated to fit the target*
  - any recognized model could be used to give sensible answers
  - answers are *independent of the choice of model*
Agenda

1. Introduction to ESGs
2. Real World vs. Market Consistent ESG
3. Basics
4. Market Consistent ESG
5. Real World ESG
6. Practical Modelling Issues
7. Q & A
What is calibration

- ESGs are implementations of time varying random variables (stochastic differential equations) with several parameters.
- Calibration is the process of choosing the parameters so that the model fits the market or the target.
Choose parameters to fit the target as well as possible
Each asset needs to be calibrated

<table>
<thead>
<tr>
<th>Asset</th>
<th>Market-consistent (example)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield curve</td>
<td>Yield curve</td>
</tr>
<tr>
<td>Interest rate volatility</td>
<td>Swaptions</td>
</tr>
<tr>
<td>Corporate bond volatility</td>
<td>Credit derivatives</td>
</tr>
<tr>
<td>Equity volatility</td>
<td>Equity options</td>
</tr>
<tr>
<td>Property volatility</td>
<td>Historic volatility – options do not exist</td>
</tr>
<tr>
<td>Exchange rates</td>
<td>Exchange rate options</td>
</tr>
<tr>
<td>Inflation</td>
<td>Index linked options</td>
</tr>
</tbody>
</table>
What have we achieved?

Model Error

Model of the Market / target

Market / target

4. Market Consistent ESG
How good are the scenarios at simulating the model

Model of the Market / target

Simulation of the Model of the Market / target

Simulation Error
Agenda

1. Introduction to ESGs
2. Real World vs. Market Consistent ESG
3. Basics
4. Market Consistent ESG
5. Real World ESG
6. Practical Modelling Issues
7. Q & A
Recap – what are real-world scenarios?

- Capture the dynamics we expect in underlying economic indicators
- Calibrated to
  - historic analysis
  - management’s best-estimates of the future
- Risk premia on risky assets
Recap – How are real world scenarios used?

- Giving probability distributions of likely future outcomes:
5. Real World Scenarios

How are real world scenarios used

- Recall: output is realistic distribution of returns
- Can be used to capture the impact of
  - Portfolio selection
  - Risk mitigation
What makes a good model?

- Good economic properties
- Internal consistency
- Adequately reflecting the data
  - Ranges
  - Relationships
- Good calibration
5. Real World Scenarios

**Good Economic Properties**

- **Short term relationships**
  - No free lunches: extra return means extra risk
  - Interest rate and inflation differentials affecting exchange rates
  - Simultaneous market panics

- **Long term relationships**
  - Economic theory describes some causes and effects over mid to long term
5. Real World Scenarios

Example of Economic Model

Short and Long Interest Rates and Full Government Yield Curve

→ Real GDP

→ Price Inflation

→ Earnings Yield
→ Earnings Growth

→ Currency Strength

→ Credit Spreads
Consistency: Internal and with Data

- What mathematical processes provide good models?
  - Good characteristics
    - Relative to the data
      - Right moments: mean, volatility and tails
      - Shape of distribution
      - Similar behaviour over different return ranges
  - Tractable
  - Behaviour over time
    - sensible over very short timeframe
    - fits the data over different time horizons
Calibration Methodology

- Focus on representing the observed characteristics of “real world” events
- Calibrated by judgement / statistical analysis
  - Current and long term conditions
  - Short term and long-term volatility
  - Mean reversion, momentum
Calibration Methodology

- Economic data highly variable
  - seldom definite answer
  - degree of judgement

- Derive statistics by examining data over a number of different time periods
  - More weight on more recent data
  - Reflect trends

- More emphasis on relationships with
  - economic underpinning
  - and/or strong statistical significance
Calibration Methodology

- Starting point: average data values
  - Can be inappropriate, e.g. where dropping or adding an extra few years has a large effect
  - Sometimes not enough historical data reduced statistical significance
  - Structural changes in market

- Need to check:
  - results give suitable range of values
  - meeting one set of statistics does not give unreasonable values for other possible statistics
5. Real World Scenarios

Assumption procedures

- Many ways to estimate the assumptions
  - Historical averages for key rates / returns
  - Consensus approach -- professional opinions
  - Econometric models
  - ‘Building block’ approach, setting long term spreads above/below an ‘anchor’ point
    - build up from assumed inflation rate
    - build up/down from long bond yield
Calibration methodology example: Equities

- Data sources – historic data on return indexes
- Main calibration drivers include
  - Short and long term volatility
  - Correlations to bond returns and inflation
  - Serial correlation
  - Skewness and kurtosis
Calibration - Warning

- Credible historic data not available for the tails of distributions, especially extreme tails
- Lots of short term data; much less independent data on long term returns
- Need another way to maximise information from the data e.g. extreme value theory
Real-world modelling - summary

- Use when trying to assess the likelihood of a future event
- There is no right answer
- Use past history and degree of judgement
- Need to have a sensible economic structure to ensure plausible results
Agenda

1. Introduction to ESGs
2. Real World vs. Market Consistent ESG
3. Economic vs. interest Rate Scenario generation
4. Market Consistent ESG
5. Real World ESG
6. Practical Modelling Issues
7. Q & A
6. Practical Modelling Issues

**Option-pricing models — some practical considerations**

- The “risk-free” expected returns in these models have moved in recent years from the government bond rate to the swap curve
  - Makes swaptions easier to price
  - Equity options generally priced off of swap curve
  - Some interest rate options still priced off of Treasuries
6. Practical Modelling Issues

**Option-pricing models — some practical considerations**

- Implied volatilities are not constant
  - Vary by maturity
  - Vary by “in-the-moneyness” — higher volatility
    - market values when strike price differs from current priced — also called volatility “smile”
  - May not be material for insurance product valuation

- Implied volatilities for long-term equity options are difficult to obtain
  - Need investment bank quote — may include margin
? - questions